

Insurance Reserve Prediction: Opportunities and Challenges

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Abstract—Predicting claims’ reserve is a critical challenge for insurers and has dramatic consequences on their managerial, financial and underwriting decisions. The insurers’ capital and their underwriting capacity of further business are subject to the unexpected reserve estimation. Increasing premium rates and adjusting the underwriting policy decisions may balance the impact of unexpected claims. Consequently, this will implicate their business opportunities negatively. Fortunately, several papers focusing on the prediction of insurance reserve have been published in the literature. In this paper, we provide a comprehensive review of the research on the insurance reserve prediction techniques in economics and actuarial science literature as well as machine learning and computer science literature. Moreover, we classify these techniques into different approaches based on the prediction mechanism they use in estimation. For each approach, we survey reserve prediction methods, and then show the similarities and differences among them. In addition, the review is armed with a discussion on main the challenges and the future opportunities.

Index Terms—Reserve prediction, Insurance data analytics, Loss estimation, Actuarial chain ladder, Stochastic methods.

I. INTRODUCTION

In the insurance domain, an insurance policy covers a defined period. If a loss occurs during the period of the policy, then the insurance company should compensate the policyholder by an amount equivalent to the loss suffered. The formal request for compensation by a policyholder is called an insurance claim and the amount requested is called the loss amount. In fact, insurance companies do not know the number and cost of claims in advance. However, they must make sure that the size of the reservoir of money they hold is adequate to cover the potential liabilities already assumed. This reservoir of money to cover future claims is known as a loss reserve. The future obligations arise naturally at a specific date; the volume and value of future claims must be accurately predicted so adequate but not excessive reserve is held.

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Insurance companies have a strong interest in the accurate estimation of loss reserve values because of the following reasons [1]; Loss reserves are considered as a loan that the insurance company owes its customers. Under-estimation of reserve values may cause a failure to meet claim liabilities. By contrast, an insurer with overestimation reserve values may result in a weaker financial position than it truly has and cause loss market share. Additionally, the reserve value is used in the insurance policies pricing process because they provide estimates of unpaid cost of insurance. Furthermore, laws and regulations require identifying loss reserve values and the public customers are interested in the financial strength of insurance companies. Moreover, many investors make decisions based the values of loss reserve values. Insurance loss reserving prediction techniques aim to estimate the final value of open claims that already happened and for which the insurer will be committed to pay [1].

II. CATEGORIZATION OF INSURANCE LOSS RESERVE TECHNIQUES

In the literature we can see that insurance reserve prediction techniques may be classified into two main categories; classical actuarial chain ladder or stochastic methods [2]–[14], and machine learning based reserve prediction methods [15]–[17] as seen in Figure1. The classical reserve analytics techniques are based on chain ladder method, which relies on building two-dimensional matrices called reserve triangles. Reserve triangles are developed from accumulating claims data over the claim lifetime. Then a stochastic process is used to generate the run-off matrices from the claim data. These run-off-matrices are used in the estimating the final reserve value. Finally, stochastic regression models are used in predicting the insurance reserve values for these claims.

The stochastic reserving approach assumes that the behavior and activities of insurance claims that happened in the past will continue to be happened in the future. In other words, there is

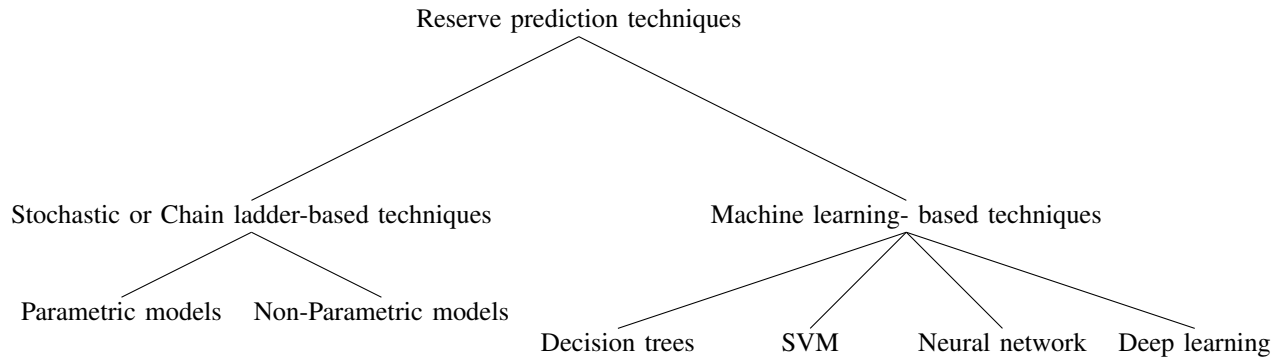


Fig. 1. The Taxonomy of Insurance Reserve Prediction Methods

no significant change in the behavior of claims' activities over time. Thus, they use simple statistical forecasting techniques e.g., regression models to predict the final value of claims. Classical reserve analytics techniques are simple as well as fast techniques. However, Avanzi et al., [2] claim that stochastic methods are not accurate compared to machine learning-based methods especially for complex claims data. The complexity of data may result from some random fluctuations in the claims data, which results in changing what will happen in the future from what has been seen in the past. Stochastic reserve analytics techniques can be further divided into two main categories [2]; parametric models and non-parametric models. The parametric models rely on numerical values in claim data following statistical distributions, and these models try to find the best parameters' values of these statistical distributions. For example, if the claim reserve values follow a normal distribution then these models aim at finding the best values of the normal distribution parameters, mean and standard deviation, to predict the reserve value in the future. While, non-parametric models do not assume any statistical distribution [18]. Non-parametric models are used when the insurance claims data fails to follow any statistical distribution. However, parametric models usually have better performance and are faster than non-parametric models but parametric models cannot be used if the insurance data does not fit into any statistical distribution. On the other hand, machine learning based techniques try to deal with complex insurance data, where the behavior or activity that gave rise to the claims in the past will not appear in the future. In other words, if insurance claims have different characteristics over time, and thus, there is no pattern (model) can represent the development of these insurance claims over their lifetime because of random fluctuations. Therefore, simple estimation models used in stochastic model and non-parametric methods for future claim reserves but not reported claim reserve estimation s e.g., chain ladder and linear regression are no longer be suitable to predict the claims' characteristics in future. Consequently, machine learning-based methods, which rely on nonlinear predictive techniques, become more promising for this kind of data.

Machine learning techniques may be further divided into four categories; decision trees-based techniques [19], support

vector machine SVM-based techniques [20], neural network based techniques [15], [17] and deep learning based techniques e.g., [16]. The machine learning based techniques have the following advantages over the classical chain ladder techniques; they allow for joint modeling of many multiple numerical values e.g., paid losses and claims outstanding. Additionally, they can incorporate multiple heterogeneous inputs formats e.g. numerical, categorical and textual data. Moreover, they can jointly handle different types of insurance claims for different types of insurance claims over different time periods in a single model. Finally, they significantly outperform existing stochastic methods in accuracy because machine learning models make use of heterogeneous claim data e.g., textual report which can not be used in classical stochastic techniques. As a result, more data could enhance the understanding and representation of prediction models, thus, they improve the prediction accuracy [16].

III. STOCHASTIC RESERVE PREDICTION METHODS

Stochastic reserve prediction methods are in widespread use throughout the actuarial literature. They are based on building loss triangles the [2]–[14]. Stochastic loss-reserve prediction methods have different taxonomies based on model parameters or number of business lines. In the model parameters, Stochastic loss-reserve prediction methods can be divided into two groups; parametric methods, where claim data can fit into statistical model or pattern then the goal is this case searching for the best values of model parameters. However, if the data can not be fitted into pattern then non-parametric loss reserve techniques could be used which is not depends on statistical distribution. On the other hand, there is another taxonomy for stochastic loss reserve methods based on number of business lines. They may be classified into two main groups; single business line e.g., [21], [22], and [15] and multiple business lines e.g., [18], [2], [23] and [19] as shown in Figure2. The single business line approach relies on handling individual business line rather than handling multiple business line jointly to focus on detailed information and behavior of individual claims [15]. However, the multiple business lines approach is more common in stochastic methods. It relies on aggregating claims data from different business lines because insurance companies usually have multiple business lines. Thus, the

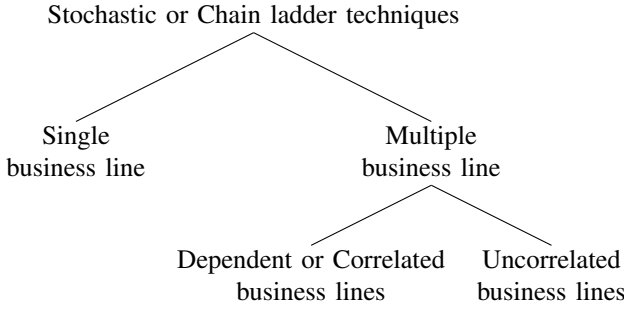


Fig. 2. The Taxonomy of Stochastic Insurance Reserve Prediction Methods

insurer aims to group the reserves of individual business lines together to enjoy the diversification benefits because the information which comes from other business lines enrich the generated model that refine the understanding of claims behavior which leads to improving of model accuracy [18]. However, the multiple approach will not get accurate results unless there are perfect dependencies among business lines [18]. Perfect dependencies means business line are strongly affect each other. Thus, the stochastic reserve prediction solutions can be classified into two categories; stochastic loss reserving with dependence and stochastic loss reserving with non-perfect dependence [2].

Examples of Stochastic loss reserving models with dependence contain [24], [25], [26], The chain ladder model and Mack's model [27] and [18]. However, examples of stochastic loss reserving models with non-perfect dependence include [28], [29], [30], and [31].

A novel stochastic loss reserving parametric model with dependence is proposed in [2] based on the statistical Tweedie distributions [32] to utilize the richness of these distribution. Tweedie distributions are a family of probability distributions containing the purely continuous distribution such as normal, gamma and inverse Gaussian distributions, the purely discrete scaled such as Poisson distribution, and the class of compound Poisson–gamma distribution [32]. Tweedie distributions are often used as generalized linear models. The proposed multivariate Tweedie approach captures cell-wise dependence; the correlation among claim values in certain dates within the loss triangles cells. The multivalued Tweedie approach can be generalized to multiple business lines. Consequently, a multivariate Tweedie approach was proposed to capture cell-wise dependence to improve insurance loss reserve techniques [2]. Moreover, the moments that are obtained analytically can be expressed in closed form. This model is based on data simulation to estimate the parameters' values. Two simulated datasets are generated to evaluate the effectiveness of the fitting procedure. The first dataset is generated from a multivariate Tweedie distribution with parameters chosen to replicate an empirical data set. While, the second experiment utilises a simulated dataset to determine the performance of the fitting procedure in presence of zeros. The experiments showed that Tweedie's compound Poisson distribution with $1 < p < 2$, is a useful subset of distributions in loss

reserving with the ability to accommodate masses at zeros [33]. Furthermore, it is evaluated using Pennsylvania National Insurance Group (Schedule P) dataset [34]. The real data experiments show that the multivariate Tweedie model has a very good fit to the data set. The results are obtained efficiently using Markov Chain Monte Carlo (MCMC) method. However, this model has the advantage of having tractable cumulants of outstanding claims to efficiently calculate the mean and the variance of total outstanding losses. Finally, the multivariate Tweedie approach [2] is a general approach that provides great flexibility with potential developments in activities leading to claims. However, this approach should require using the same power parameter p for all lines of business as well as perfect dependence structure for all business lines [2].

Another reserve parametric model with dependence is a censored Copula model [35]. It is based on using the Coupla; an actuarial multivariate cumulative distribution function where the marginal probability distribution of each variable is uniform. A censored means estimating the evolution of open insurance claims. This model relies on the dependency structure of age (the time before settlement) and value of insurance claims. This model targets open (censored) claims. Uncensored claims tend to be small claims because they usually take short time to close. This model extends the generalized linear regression model to the context of censoring. It handles difficulty of estimating a conditional copula function without having especially in high dimensional data [35].

The Kaishev's model has been proposed [23] to model dynamic operational risk capital within multiple business line. The operational reserve loss and reserving strategy are significantly affected by of the sizes of operational losses. Furthermore, the authors suggest studying the crossover dependence (the correlation) between inter - occurrence times of losses and their amounts. The authors suggest studying the crossover dependence (the correlation) between inter-occurrence times of losses and their amounts [23]. Kaishev model relies on employing the finite time probability (operational risk measure) within a general risk model because the finite time probability-based model can handle non-homogeneous operational loss frequency and dependent loss severities, which may have any joint discrete or continuous distribution. This article use Loss Data Collection Exercise (LDCE) dataset to test this model [23].

IV. MACHINE LEARNING BASED TECHNIQUES

As we mentioned before the chain ladder model assumes that the behavior and activities that give rise to insurance claims that happened in the past will continue to be happened in the future. An example of a behavior or activity would be careless driving causing accidents or a hurricanes causing property damage. So the model is assuming that the number of car accidents and hurricanes that occurred in the past will continue into the future. The chain ladder model and the assumptions in Mack's model [27] are not always the most appropriate to use in practice. Thus, the machine learning approach is more promising. This approach focuses

on using complex (nonlinear models) prediction techniques to estimate insurance loss reserve values. It aims to improve prediction accuracy based on including claim information e.g., business line, claim type and age in prediction process. Machine learning techniques may be categorized into four categories; decision trees-based techniques e.g., [19], SVM-based techniques e.g. [20], neural network based techniques e.g., Neural Networks based Chain-Ladder Reserving (NNCLR) [17] and deep learning based methods e.g., Deep Triangle [16]. A decision tree – based technique was proposed [19] to predict insurance reserve values. It is considered a multi-label supervised learning approach, where each claim is categorized into one of a number of well-predefined labels. This approach consists of two phases; two phases: The training phase and the testing phase. In the training phase, well-known samples of insurance claims from all classes are required in building a classification model, which is used to distinguish among the behavior of each class. The established classification model is then used for predicting the class label for the open claims in the testing phase [17].

In contrast to the classical chain ladder approach, this technique uses all available information about claims to build a decision tree. It is considered a data driven technique, which means that the established decision tree affected by the data in the training phase e.g., claim samples in the training phase should cover all classes with similar probability of occurrence. It deals with the data heterogeneity, different data types e.g., categorical and numerical with different format. The data heterogeneity is resulting from the insurance claims evolution and/or their characteristics. A decision tree – based technique [19] relies on developing a weighted CART algorithm (Classification And Regression Trees) [36]. A decision tree–based approach [19] focus on structured data, numerical and categorical attributes. This model aims at predicting insurance claims, which have been reported and not yet settled (RBNS) and open claims. Furthermore, it is promising because it is simple and easy-to-understand. Additionally, it is based on theoretical guarantees and consistent procedure because its idea is clear which is based on consists of recursive splitting the claims into more homogeneous groups. Moreover, it is can handle long-development insurance claims that claims spread over time and the model has discriminating power of covariates. However, it is not very robust towards the change in the patterns of emerging insurance claims [15] because the change in data may require the tree be re-built from scratch. In the future work, the authors suggest using bootstrap resampling technique to get some confidence interval around the estimation. A decision tree–based approach uses a pruning strategy to derive consistency results and for the selection of an optimal subtree. A simulation experimental study as well as real data are used to evaluate the proposed technique [19]. This model is evaluated using real individual claim dataset for RBNS and closed claims.

Neural Networks based Chain-Ladder Reserving (NNCLR) [17] model extends the classical chain ladder techniques but it relies on using nonlinear neural network regression mod-

els instead of linear regression models. NNCLR focuses on employing claims' information e.g., business line, claim type and age in the prediction process. (NNCLR) [17] starts with building Mack's Chain Ladder [27], and then it preprocess the data for shallow neural network. The preprocessing process consists of two tasks; transforming categorical attributes to numerical features to be suitable for neural network and all numerical features should have the same scale. In transforming categorical attributes to numerical attributes, NNCLR replace categorical values by binary values using dummy variables. It transforms a categorical attributes that contains r categories into $r-1$ binary variables [37]. Then, all numerical attributes are normalized using min-max normalization method and hence all numerical feature have the same range of values in order to have the same importance in the model. Finally, NNCLR creates a simple neural network to forecast the reserve values. It aims to improve the prediction accuracy by accurate description of claims information utilizing individual claims feature information. It considers feature information as static features because in particular, NNCLR does not change over time.

A more recent machine learning-based insurance reserve prediction algorithm is Deep Triangle [16]. It proposes loss reserving based on deep neural networks. It consists of two steps; the first step is based on building loss triangle then it uses recurrent deep neural network based on loss triangle to predict loss reserve values. Deep Triangle is a multi-task network with multiple prediction goals depending on number of objectives models. The original deep triangle techniques are designed for two objectives; paid losses and claims outstanding. Thus the proposed Deep Triangle has two prediction goal one for paid loss and the other prediction goal for claims outstanding. In addition, it can incorporate heterogeneous inputs (different format, and time stamp). It is appropriate for loss reserving data across multiple lines of business. Deep Triangle builds one model for each line of business and each model is trained on data from multiple companies [16].

The deep learning framework for estimating the paid losses is able to attain performance comparable other stochastic reserving techniques without expert parameters. It can incorporate multiple heterogeneous inputs and train on multiple objectives simultaneously, and it can customize of models based on available data. It takes longer time in processing. It requires minimal feature engineering and expert inputs. Thus, this model may be automated to produce forecasts more frequently than manual workflows. Insurance information is represented as sequential data. Thus, the most appropriate kind of deep neural network is Recurrent Neural Network (RNN) [15]. The recurrent neural network (RNN) is a common type of artificial neural networks where connections between nodes form a directed graph along a temporal sequence. This allows it to exhibit temporal dynamic behavior. Therefore, Deep Triangle relies on using a sequence-to-sequence architecture recurrent units (GRU), which is a type of RNN building block [38].

V. DISCUSSION

The prediction of reserve value is the most important problem in the field of insurance data analytics. It aims at forecasting the final value of insurance claims. The insurance reserve prediction techniques are classified into two main categories; Stochastic and machine learning-based techniques.

The Stochastic or chain ladder approach aims at predicting accurate reserve based on the compacted data or reserve triangles. Additionally, it is easy to implement and understand. However, it is based the assumption that the data and activities will occur similarly in the future as its occurrence in the past. Thus, it will have bad performance if the claim pattern change over time, which deviates from the underlying assumption.

On the other hand, the machine learning approach relies on using information about claim in the prediction process e.g., categorical and textual data. The claim information is very useful in machine learning approach because it improves its performance. Machine learning based insurance reserve techniques are more complex than chain ladder techniques. Company stakeholders, particularly actuarial peoples have difficulty in understanding, developing, applying and putting these techniques in practice [19]. By contrast, and despite the widespread availability use of stochastic techniques in practice, machine learning-based insurance reserve value prediction are more promising and having accurate prediction rate compared to classical stochastic insurance reserve analytics techniques. Most machine learning approaches; decision trees, neural network and deep neural network techniques obtained better results than stochastic techniques.

VI. CONCLUSION AND FUTURE WORK

Predicting the loss reserve value is very significant problem in the insurance sector. Available methods for the estimation of loss reserve value in the actuarial as well as the machine learning literatures are reviewed. The reserve prediction techniques have been classified into different approaches based on the prediction mechanism, focusing on the primary similarities and differences. The review is armed with a discussion on main the challenges. There are several directions for future works in prediction of loss reserve value such as the following directions:

- Enhancing the data preprocessing should process improve the estimation process to increase data quality. Examples of preprocessing tasks handling missing values, handling noisy data, and better handling of categorical data correlation/association.
- Improving the model prediction power though feature selection and dimensionality reduction features selection or reduction will be promising to improve the processing and accuracy of reserve prediction methods
- Utilizing other machine learning prediction approaches, e.g., Naïve Bayesian classifier, knn, and random forest.
- Incorporating the unstructured data such as insurance reports, social behaviour data, spatio-temporal data, and image data & multimedia data.

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